

Beam–plasma interaction in strongly coupled plasmas

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2006 J. Phys. A: Math. Gen. 39 4613

(<http://iopscience.iop.org/0305-4470/39/17/S47>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.104

The article was downloaded on 03/06/2010 at 04:25

Please note that [terms and conditions apply](#).

Beam–plasma interaction in strongly coupled plasmas

Marlene Rosenberg¹, Gabor J Kalman², Stamatios Kyrkos^{2,3}
and Zoltan Donko⁴

¹ Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093, USA

² Department of Physics, Boston College, Chestnut Hill, MA 02467, USA

³ Department of Chemistry and Physics, Le Moyne College, 1419 Salt Springs Rd, Syracuse, NY 13214, USA

⁴ Research Institute for Solid State Physics and Optics, Hungarian Academy of Sciences, POB 49, H-1525 Budapest, Hungary

E-mail: rosenber@ece.ucsd.edu

Received 30 September 2005, in final form 5 December 2005

Published 7 April 2006

Online at stacks.iop.org/JPhysA/39/4613

Abstract

The well-known problem of beam–plasma instability acquires new aspects when one or both of the two components (the beam and the plasma) are strongly interacting. We have now theoretically considered the case when the plasma is in the solid phase and forms a lattice. In this situation, the inherent anisotropy of the lattice leads to a coupling between the longitudinal and transverse polarizations. One of the novel features of the beam–plasma instability in this scenario is the possible excitation of transverse modes, which should be an experimentally observable signature of the instability. We have initially concentrated on a 2D toy model with the beam lying in the lattice plane. At the same time, we have initiated a molecular dynamics simulation program for studying various aspects of the penetration of a beam into a plasma lattice. The beam parameters can be adjusted in order to see the effects of increasing coupling strength within the beam and to distinguish between collective phenomena and scattering on individual particles. When both components are strongly interacting, a number of remarkable phenomena—trapping of beam particles, creation of dislocations, local melting of the lattice—may be observed.

PACS numbers: 52.27.Gr, 52.27.Lw, 52.35.Qz

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In this paper, we examine how beam–plasma instabilities develop in strongly coupled plasmas, in particular when the plasma is in the crystalline state, in order to motivate related experiments in dusty plasmas. A beam of light particles (e.g. ions) penetrating a dusty plasma, or two inter-penetrating dusty plasmas, are expected to generate a beam–plasma instability [1, 2]. In a Vlasov plasma, beam–plasma instabilities result in the excitation of longitudinal plasmons. In a strongly coupled plasma, transverse (shear) waves develop, and under certain circumstances one can expect the excitation of transverse waves as a result of a beam–plasma instability [3]. This is because the phonon eigenmodes in the crystalline solid state in general have a mixed longitudinal–transverse polarization.

The plasma system we wish to analyse reflects the properties of a complex plasma in a crystalline state, consisting of highly charged mesoscopic grains, immersed in a background of a polarizable electron–ion plasma; the effect of this latter is assumed to be restricted to modifying the grain–grain interaction to a screened, Yukawa type interaction. We approach the issue in several stages. First, we establish a collective coordinate formalism, appropriate for the analysis of a beam penetrating into a lattice. Next, we construct a two-dimensional toy model amenable to analytic treatment, and show that excitation of transverse waves can occur indeed, with growth rates comparable to longitudinal growth rates. Finally, computer simulation results highlight the various scenarios that can take place in beam–lattice interactions.

2. Analytical and numerical results

In order to analyse the combined beam–lattice system, a common language appropriate for both systems has to be established. In a lattice, particle i is displaced from its equilibrium position x_i^μ by ξ_i^μ . Phonons are conventionally described in terms of the collective coordinates $\xi_{\mathbf{k}}^\mu$ defined by

$$\xi_i^\mu = \frac{1}{V} \sum_{\mathbf{k}} \xi_{\mathbf{k}}^\mu e^{i\mathbf{k}\cdot\mathbf{x}_i}. \quad (1)$$

A similar language can be adopted for the beam, by defining a beam collective coordinate

$$\eta_i^\mu = \frac{1}{V} \sum_{\mathbf{k}} \eta_{\mathbf{k}}^\mu e^{i\mathbf{k}\cdot\mathbf{x}_i}. \quad (2)$$

Assuming that (i) the interaction within the beam and between the beam and the lattice is weak and that (ii) the beam couples to the longitudinal mode only, the fundamental equation for the system is (note that the interaction potential has not yet been specified)

$$\begin{aligned} \omega^2 \xi_{\mathbf{k}}^\mu &= \left\{ \Omega_0^2(k) \frac{k^\mu k^\nu}{k^2} + D^{\mu\nu}(k) \right\} \xi_{\mathbf{k}}^\nu + \omega_0^2(k) \frac{k^\mu k^\nu}{k^2} \eta_{\mathbf{k}}^\nu \\ (\omega - \mathbf{k} \cdot \mathbf{v})^2 \eta_{\mathbf{k}}^\mu &= \Omega_0^2(k) \frac{k^\mu k^\nu}{k^2} \xi_{\mathbf{k}}^\nu + \omega_0^2(k) \frac{k^\mu k^\nu}{k^2} \eta_{\mathbf{k}}^\nu. \end{aligned} \quad (3)$$

Here $\Omega_0(\mathbf{k})$ is the frequency of the Vlasov (RPA) plasma mode in the unperturbed system (with no beam), $\omega_0(\mathbf{k})$ is the Vlasov plasma mode in the beam, \mathbf{v} is the beam velocity, and $D^{\mu\nu}(k)$ is the lattice dynamical matrix. In the unperturbed system, the quasi-longitudinal and quasi-transverse phonons are denoted by $\Omega_L(\mathbf{k})$ and $\Omega_T(\mathbf{k})$, respectively. The growth rates δ_L and δ_T are provided by the solution of equation (3) in the vicinity of the unperturbed phonon frequencies

$$\omega_m(\mathbf{k}) \approx \Omega_m(\mathbf{k}) + \delta_m, \quad (4)$$

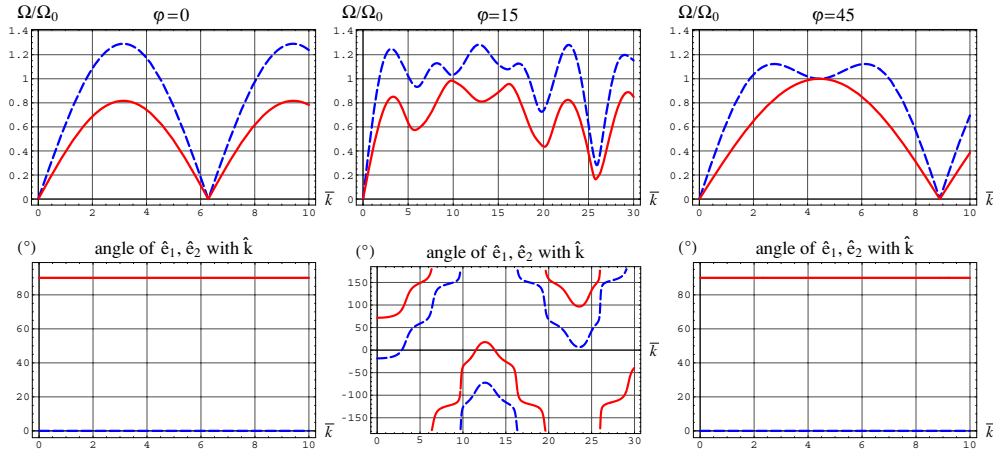


Figure 1. Representative plots for phonon dispersions and polarizations. Dashed lines correspond to quasi-longitudinal and solid lines to quasi-transverse modes, respectively. $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ are the polarization vectors. The numerical analysis was done for $\alpha = \frac{1}{4}$ and $\gamma = \frac{1}{12}$.

where $m = L, T$ for the two modes respectively, together with the beam–plasma resonance condition

$$\omega - \mathbf{k} \cdot \mathbf{v} \approx \delta_m. \quad (5)$$

We have analysed a simple model, originally devised by Montroll [4], that provides an analytical expression for the phonon dispersions. The model comprises a two-dimensional square lattice with principal axes along \mathbf{x} and \mathbf{y} , and considers nearest neighbour and next-nearest neighbour interactions only (with respective coupling constants α and γ , where $\gamma < \alpha$). We refer to this as a ‘toy model’ which may represent qualitatively a Yukawa system in the limit of large screening, noting that the full Yukawa interaction and the actual hexagonal geometry of a Yukawa lattice would affect quantitative results. The quasi-longitudinal and quasi-transverse modes are given by

$$\Omega_{L,T}^2(k, \phi) = \frac{D_{11} + D_{22}}{2} + \Omega_0^2 \pm \frac{1}{2} \sqrt{D_{11}^2 - 2D_{11}D_{22} + D_{22}^2 + 4D_{12}D_{21}}, \quad (6)$$

where

$$D_{11,22} = A_{1,2} \cos^2 \phi + A_{2,1} \sin^2 \phi \pm 2B \sin \phi \cos \phi,$$

$$D_{12} = D_{21} = (A_2 - A_1) \sin \phi \cos \phi + B(\cos^2 \phi - \sin^2 \phi), \quad (7)$$

$$A_{1,2} = -2\alpha(1 + \cos \bar{k}_{x,y}) + 4\gamma(1 - \cos \bar{k}_x \cos \bar{k}_y), \quad B = 4\gamma \sin \bar{k}_x \sin \bar{k}_y$$

$\Omega_0^2 = 4\alpha$, ϕ is the angle between \mathbf{k} and the \mathbf{x} axis, and $\bar{k} = ka$, where a is the lattice constant. Figure 1 shows representative results. Note the qualitative resemblance to the 2D Yukawa phonon dispersion [5]. (For a $\kappa = 1$ Yukawa screening parameter, the ratio α/γ would be ≈ 4.5). The beam is taken to be in the plane of the lattice and its directed velocity is in the \mathbf{x} direction. We assume the beam particles are weakly coupled to each other and to the lattice. The motion of the lattice particles is determined by all the microscopic forces acting on them and thus includes both the effects of collisions between charged particles and of their collective interactions. However, collisions with neutral particles are not included at this stage. The frequencies $\Omega_m(k, \phi)$ and wave numbers $k_m(v, \phi)$ ($m = L, T$), in the vicinity of which the instabilities are generated, are determined by the beam resonance condition

$$\Omega_m(k, \phi) = kv \cos \phi. \quad (8)$$

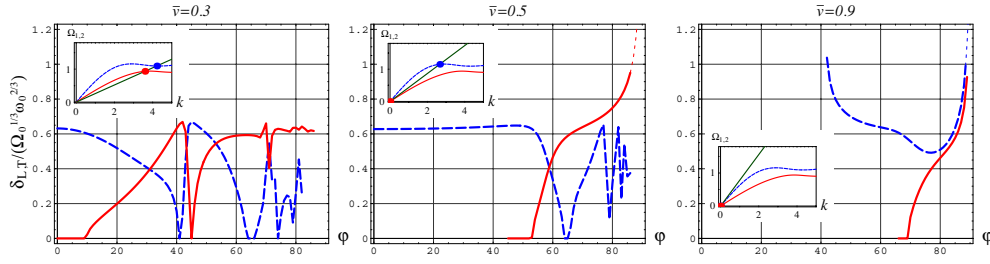


Figure 2. Longitudinal (dashed) and transverse (solid lines) growth rates for three different beam speeds $\bar{v} = v/(\Omega_0 a)$. ($\alpha = \frac{1}{4}, \gamma = \frac{1}{12}$). If v is higher than one of the sound speeds, there are angles for which the resonance condition (8) is not satisfied. The insets show the resonance condition for $\phi = 30^\circ$.

Table 1. The parameters used in the MD simulations of figure 3. Subscripts b and p stand for ‘beam’ and ‘plasma’, respectively.

ν	μ	ζ	f	h	u	w
n_b/n_p	M_b/M_p	Q_b/Q_p	$v/(\Omega_p a)$	$\frac{M_b v^2}{2} / \frac{Q_p Q_b}{a}$	$\left(n_b \frac{M_b v^2}{2}\right) / \left(n_p \frac{Q_p Q_b}{a}\right)$	ω_p / Ω_p
0.001	10	1	3	90	0.09	0.002
0.1	0.1	0.1	3	9	0.09	0.18

The relationship between the beam speed v and the longitudinal and transverse sound speeds s_L, s_T determines the qualitative behaviour of the growth rates. The longitudinal and transverse growth rates depend on the respective frequencies and thus ultimately, for a given beam speed and direction, there are two growth rates $\delta_L(v, \phi)$ and $\delta_T(v, \phi)$ (figure 2). The longitudinal and transverse growth rates are of the same order, but the transverse instability generally appears for larger angles. For beam speeds between the longitudinal and transverse sound speeds, the transverse instability could be more important, because it appears at lower k values.

3. Computer simulations in 2D

Molecular dynamics simulations can provide a valuable insight into the behaviour of a plasma excited by a particle beam [6]. In our MD simulation, particles are assumed to interact via the Yukawa potential, with $\kappa = 1.0$. At the initialization of the simulation, the plasma particles are arranged in a hexagonal lattice with a nearest neighbour distance set to $1 \mu\text{m}$. Beam particles of given (i) number, (ii) charge, (iii) mass and (iv) velocity are injected from one side of the simulation domain (at $x = 0$ and from random positions in the perpendicular direction). Their subsequent motion is determined by the interaction with the plasma particles. When a beam particle leaves the simulation domain, a new beam particle is again injected at $x = 0$ in order to keep the number of beam particles constant. It should be noted that, in contrast to the theoretical model, here the computer code includes the full electrostatic interaction between all, i.e., beam and plasma, particles. By choosing different density, charge and mass ratios between the plasma and beam particles, different conditions of the beam–plasma interaction are realized; e.g. massive and high-velocity beam particles can be observed to pass through the simulation domain without significant momentum transfer, while light particles may be trapped in the potential minima of the plasma lattice.

The simulations have been performed for various combinations of parameters, shown in table 1, namely ν , the ratio of the beam to plasma density, μ , the ratio of the masses

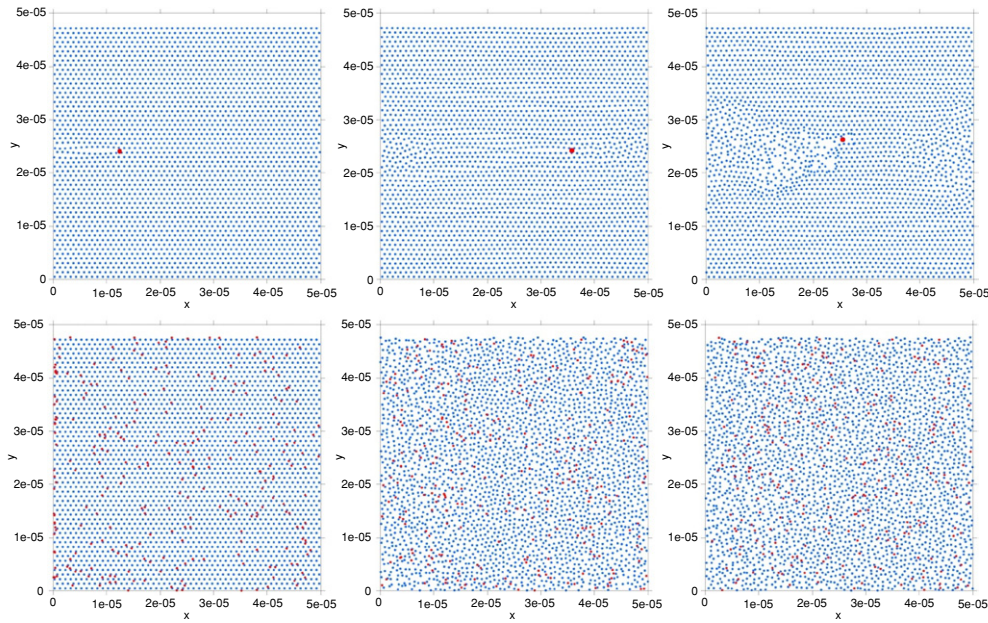


Figure 3. Two examples of MD simulations. Each row shows three snapshots of a supersonic ($f = 3$) beam incoming from the left. In the first row, the snapshots were taken at times $\omega_p t = 15.9, 110.9$ and 232.5 , respectively. In the second row, the times are $0.2, 42.4$ and 118.5 . The interparticle distance for the plasma before the impact of the beam is $1 \mu\text{m}$, and the units of the x and y axes are given in metres.

of the beam and plasma particles, and ζ , the ratio of the beam to plasma particle charge, as well as the ratio f of the beam speed to the sound speed, that determines whether the flow is supersonic ($f > 1$) or subsonic ($f < 1$). In figure 3, the sequence of snapshots on the top shows the waves and the subsequent disorder generated in the wake of a single massive beam particle with supersonic speed. The bottom sequence shows the turbulence generated by the passage of a beam of light particles with density amounting to 10% of the density of the plasma. While collective effects seem to be prevalent in both cases, it is difficult in general to separate them from the effect of two-particle collisions without further study.

4. Conclusions

When the plasma is in the crystalline solid state, the character of the beam–plasma instability (weak beam penetrating into the lattice) changes: due to the mixed polarizations of the lattice phonons, both longitudinal and transverse instabilities are generated. Since the longitudinal and transverse sound velocities are different, the beam velocity can be tuned to a value where for low k values (most resilient to damping) only the transverse instability is excited. Large amplitude transverse waves seem to be more efficient in creating disorder in the lattice or melting it than the longitudinal ones. The more general case, when two strongly coupled plasmas penetrate into each other, requires more theoretical work for better understanding and experimental diagnostic tools to separate effects due to particle–particle scattering from those due to collective instabilities.

Acknowledgments

This work has been partially supported by DOE Grant no DE-FG02-04ER54804, NSF Grant no PHYS-0206695, DOE Grant no DE-FG02-03ER54716, Grant nos OTKA-T-48389 and MTA/OTKA-90/46140.

References

- [1] Lampe M, Joyce G and Ganguli G 2005 *IEEE Trans. Plasma Sci.* **33** 57
- [2] Golden K I, Kalman G J and Hammerling P 1980 *Phys. Lett. A* **80** 149
- [3] Kalman G J and Rosenberg M 2003 *J. Phys. A: Math. Gen.* **36** 5963
- [4] Montroll E 1947 *J. Chem. Phys.* **15** 575
- [5] Sullivan T, Kalman G J, Kyrkos S, Bakshi P, Rosenberg M and Donko Z this volume
- [6] Morozov I V, Norman G E, Valuev A A and Valuev I A 2003 *J. Phys. A: Math. Gen.* **36** 8723